—DRAFT—
CAN A MANROOT TENDRIL PULL ITS VINE CLOSER TO A SUPPORT EVEN AS THE TENDRIL INCREASES IN LENGTH?

MICHAEL RAUGH

ABSTRACT. It appears that a tendril can contract in span while coiling—even as it expands in length—a proposition that sounds paradoxical. But is it true? It's possible the tendril could increase in length rapidly enough to offset span contraction. The best way to resolve this would be to make measurements of real tendrils during growth. Another way is to analyze a photograph of a tendril to fit the photographed tendril to a right-circular helix. The latter method is followed here, validating the "paradox"—a tendril growing in length certainly can pull its vine closer to a support, one of its principal functions according to Charles Darwin (the other being spring-action to avoid detachment in stiff winds).

The *span* of a coiled tendril is the straight-line distance between its two ends; its *length* is the total distance around its coils, the distance a ladybug would have to walk from one end to the other. It is obvious from Fig. (1) that the span of the tendril in the photograph is much shorter than its full length if rolled out straight.[1] But it is not obvious that, if the tendril increased in length during its coiling process, its span would shrink. So we ask the question, what if the tendril grows during coiling? Can the span contract simultaneously?

We can answer the question by making a straightforward analysis of a photograph of a coiled tendril. Our analysis will show that a tendril is capable of pulling its vine from 50–70% closer to its support than it was at the moment the tendril attached to the support.

We do this by matching the tendril to a similar right-circular helix. Such a helix can be expressed in xyz-space as,[2]

Date: April 11, 2010.

[1]Rollout on a tightly coiled tendril like the one in the photograph would require inelastic strain and break tissue.

[2]See for example Do Carmo [2] or Struik [4].

$$H(\theta) = (a\cos\zeta, a\sin\zeta, \left(\frac{b}{2\pi}\right)\zeta, \quad a > 0,\ b \geq 0,\ \zeta \in [0, \infty) \tag{1}$$

where the angle ζ is measured in radians counterclockwise from the positive x-axis. Note that the tendril moves upward b units each full revolution of the tendril.

Copyright ©2010 Philip Grote.

FIGURE 1. A tightly coiled tendril. The sawtooth profile of this tendril permits an easy estimate of the ratio of the *span* of its coils to the *length* wound up in its coils. See text for sample calculation.

Let's do some mathematical surgery to compute the ratio of the span of this helix to its length. If you picture the tendril wrapped around a right-circular cylinder, and imagine a vertical cut beginning at $(a, 0, 0)$, then unwrap the cylinder and lay it flat, you can see that each revolution of the helix forms a right-triangle with legs of lengths $2\pi a$ and b. This implies that the length of one full revolution of the helix is $\sqrt{4\pi^2a^2 + b^2}$, and so for this helix the ratio of span to length of the helical segment is,

$$\frac{\text{Span}}{\text{Length of helix}} = \frac{b}{\sqrt{4\pi^2 a^2 + b^2}} \tag{2}$$

Now let's examine Fig. (1) and fit a helix to it. This is easy if we print the figure on a piece of paper and estimate the relative distance between the ends of a full coil and its radius.

To do this, I approximate any complete loop of the tendril by a loop of a co-axial helix. Since the tendril has volume and the helix does not, I fit the helical loop midway in the volume of the tendril. I don't bother with the reversal near the midpoint or the extreme ends. Of course I'm making an assumption about the symmetry of the tendril like the one in the figure, namely that viewed end-on the tendril would appear more-or-less symmetrical about its central axis, not oval or flattened. Observation of tendrils in nature confirms this is reasonable.

Choosing loops at random for measurement, I get the approximate relation $b \approx 2a$. Slight variations in this estimate will make only slight differences in the results. Substituting this relation into the ratio equation,

$$\frac{\text{Span}}{\text{Length of helix}} \approx \frac{1}{\sqrt{\pi^2 + 1}} \approx 0.3$$

Since this result holds for each loop, it implies that the final span of a tendril may be as little as approximately 30% of its mature length. This has an important consequence for us. Darwin [1, 53–55, 59] claimed that a tendril acquires the power of coiling when near its full length but loses the power when full grown if unattached. So let us suppose that a tendril could attach to a support by the time it had achieved something like 60% of its full length.[3] If that number is correct, then we can infer that a tendril is capable of pulling the vine to which it is attached at least 50% closer to the support than the vine was at the time of attachment.

This contraction of a tendril can be modulated by external constraints that prevent closure, such as intervening obstacles or other tendrils pulling in opposing directions. Another cause of weak coiling can be that the tendril was nearly senescent at the time of attachment.

[3] Darwin also asserted that tendril coiling is a consequence of differential growth on the dorsal and ventral sides of the tendril. An elegant study using finite element analysis illustrates how this is possible [3].

We have shown examples in the Gallery of tendrils that do not coil tightly—or even coil at all—presumably for such reasons.

But the fact that some tendrils do contract tightly demonstrates that they are capable of pulling their vines a fair distance closer to their supports. This they can do "paradoxically" while actually increasing in length.

References

1. Charles Darwin, *On the movements and habits of climbing plants*, NYU Press, 1989,
(Originally published on September 1, 1865 as a 118 page treatise by the Linnean Society, it was republished with an appendix and corrections in 1882; it has been published in a scholarly edition of the complete scientific writings of Darwin by NYU Press and is available elsewhere in inexpensive book format.).
2. Manfredo P. Do_Carmo, *Differential geometry of curves and surfaces*, Prentice Hall, Englewood Cliffs, 1976.
3. Annika Eberle, Kenny Quinn, and Lori Bassman, *Free coiling in tendril-bearing plants*, 6th Plant Biomechanics Conference Cayenne, November 16–21, 2009.
4. Dirk J. Struik, *Lectures on classical differential geometry*, 2 ed., Dover Publication, New York, 1988.

E-mail address: michael dot raugh (at) gmail dot com
URL: www.mikeraugh.org