Mathematical Misnomers:

Hey, who really discovered that theorem!

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Who was buried in Grant’s tomb?

Ulysses S. Grant, of course!

These are true too:

Sandwiches were invented by the Earl of Sandwich.

And Cramer’s rule was really discovered by Cramer.

But not all things are as they seem....
Kepler (b1471) discovered the planetary ellipses.

But who first thought of the inverse-square law?

Robert Hooke in letters to Newton, circa Dec. 1679.

Newton (at age 37) wrote that a dropped ball falling to the center of the Earth would wind in a spiral.

Hooke said, No, and guessed an ellipse, tugged by an inverse-square law—but offered no theory!
**The hard part was....**

- To derive planetary motion—from first principles!
- *That’s what Newton did...*
  - by first formulating the laws of mechanics and inventing his version of calculus called *fluxions*.
- Newton *proved* that elliptical orbits imply an inverse-square law, and vice-versa.
- But he didn’t formulate and solve the differential equations of motion—Johann Bernoulli (b1667) did that!
“Jakob Bernoulli’s” summation formula

\[ 1^k + 2^k + \cdots + n^k = \]

\[ \frac{n^{k+1}}{k+1} + \frac{n^k}{2} + \frac{1}{2} \binom{k}{1} B_2 n^{k-1} + \frac{1}{4} \binom{k}{3} B_4 n^{k-3} + \cdots + B_k n, \quad n > 1 \]

Essentially discovered decades earlier by Johann Faulhaber, the “Calculating Wizard of Ulm.”

The \( B \)s were named “Bernoulli numbers” by Euler — some say by de Moivre:

\[ B_1 = -\frac{1}{2}, \quad B_2 = \frac{1}{6}, \quad B_4 = -\frac{1}{30}, \quad B_6 = \frac{1}{42}, \ldots \]

\[ B_3 = B_5 = B_7 = \cdots = 0 \]
Euler’s use of Bernoulli numbers

Euler became famous in his twenties by solving the “Basel problem” (posed by Mengoli in 1644)

\[
\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \pi^2 = \frac{\pi^2}{6}
\]

Later he generalized

\[
\frac{1}{1^{2k}} + \frac{1}{2^{2k}} + \frac{1}{3^{2k}} + \cdots = (-1)^{k+1} \frac{2^{2k-1} \pi^{2k}}{(2k)!} B_{2k}
\]

And derived a generating function for the Bernoulli numbers

\[
\frac{x}{e^x - 1} = B_0 + \frac{B_1}{1!} x + \frac{B_2}{2!} x^2 + \frac{B_4}{4!} x^4 + \cdots
\]
Euler tried but failed to find simple formulas for series of odd powers, like

\[
\frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} + \cdots
\]

And nobody else has ever succeeded.

But he did find formulas for alternating series of odd powers, like

\[
\frac{1}{1^5} - \frac{1}{3^5} + \frac{1}{5^5} - \frac{1}{7^5} \pm \cdots
\]
Jacob Bernoulli’s probability theory (Ars Conjectandi)

The probability of $k$ heads in $n$ tosses is

$$\binom{n}{k} p^k (1 - p)^{n-k} \quad \text{(Bernoulli’s distribution)}$$

What does Bernoulli’s distribution look like for large $n$?

To figure it out you need to estimate

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

for large $n$ and $k$. 
Getting to “Gauss’s” normal distribution:  
*The bell curve*

In Bernoulli’s distribution, substitute “Stirling’s” formula

\[ m! \approx \sqrt{2\pi} \ m^{m+\frac{1}{2}} e^{-m} \]

Re-scale Bernoulli’s distribution (horizontally and vertically) to obtain the limiting distribution:

\[ \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \text{ (Normal distribution)} \]

de Moivre discovered “Stirling’s” formula on his way to discovering “Gauss’s” normal distribution.
How did de Moivre do it?

To derive “Stirling’s” formula, de Moivre used “Maclaurin’s” summation formula: if \( a_j = f(j) \) for differentiable \( f(x) \), then

\[
\sum_{j=0}^{n} a_j = \int_{0}^{n} f(t) dt + \frac{1}{2} f(n) + \sum_{j=1}^{m} \frac{B_{2j}}{(2j)!} \left[ f^{(2j-1)}(n) - f^{(2j-1)}(0) \right] + R_m
\]

The \( B \)s are the Bernoulli numbers again!

The summation formula was found first by Euler, so it is now called Euler’s summation formula or the Euler-Maclaurin summation formula.
So, who invented calculus?

Newton and Leibniz, right? Well, yes, but ....

Newton:
De methodis serierum et fluxionum, 1670-71
Philosophiae Naturalis Principia Mathematica, 1687
Newton used geometric methods and fluxions, which we do not use.

Leibniz:
Nova methodus..., 1684

Don’t forget Eudoxus and Archimedes!
Differential and integral calculus—as we know it

L’Hospital’s book *Analysis of The Infinitely Small, 1696* (written Johann Bernoulli!) popularized Leibniz’s approach to differential and integral calculus.

In that tradition **Euler** developed calculus in terms of *functions, infinite series, differential equations, calculus of variations*, and in laying foundations for *analytic mechanics of solids, fluids and elastic media*....

**Modern rigor came later** in the work of Cauchy (b1789), Weierstrass (b1815), Riemann (b1826), Dedekind (b1831) and Cantor (b1845).
And, Oh, yeh,

L'Hospital’s rule was discovered by Johann Bernoulli!
\[ \int_{\partial S} \mathbf{F} \cdot \mathbf{t} \, ds = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, dS \]

It's not really Stokes's theorem!

It was discovered by his friend, the Scotch-Irish physicist William Thompson—knighted Lord Kelvin.

Stokes posed it as a problem on a famous Cambridge Math contest—the *Tripos*. 
Closing thought

If you happen to be a discoverer who’s name is forgotten, you will have had all the fun making the discovery anyway.

Isn’t that the best part?
Recommended reading

Easy reads:

- *Euler, Master of us All* by Dunham
- *A Very Short Introduction to Newton* by Iliffe
- *The Calculus Gallery* by Dunham

More on methods of Newton, Leibniz and Euler:

- *Reading the Principia* by Guicciardini
- *Theory and Application of Infinite Series* by Knopp
- *Basic Calculus from Archimedes to Newton* by Hahn